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The improved surface gradient method for flows simulation in variable bed topography channel using TVD-MacCormack scheme

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SUMMARY

This paper reports four different approaches to discretize the source terms for the simulation of one-dimensional open-channel flows with rapidly varied bottom topography using TVD-MacCormack scheme. Compared with other high-resolution shock-capturing schemes, MacCormack-type predictor–corrector method is easy to implement and does not present any additional difficulty in dealing with the source terms. To avoid the generation of artificial numerical waves, if the bottom topography shows strong variation, special treatment of the source terms is still required to eliminate or reduce the artificial numerical error caused by adding TVD corrections to the method. The computed results demonstrated that the improved surface gradient method is more suitable for simulating open-channel flow with highly irregular bed topography by using the surface gradient instead of the depth gradient for TVD corrections and considering the balancing of the source terms and the flux gradients. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: variable bed topography; open-channel flow; TVD-MacCormack scheme; improved surface gradient method

1. INTRODUCTION

The prediction of shallow water flows with abrupt changes, such as hydraulic jumps, bores and surges are of great interest to hydraulic engineers. The variations of water depths and velocities in these extreme events are important parameters for the design of hydraulic systems and for flood control operations. However, due to the strong gradients inherent in the problems, most of the traditional simulation models display spurious oscillations at the shock fronts.

In the past 15 years much effort has been made in the numerical solution of hyperbolic systems of conservation laws, starting with the field of gas dynamics [1, 2]. With the advances

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made in the computational techniques, the high-resolution shock-capturing schemes have been successfully applied for solving the homogeneous form of the shallow water equations by many researchers [3–10]. However, these numerical models restrain the oscillatory behaviour in the solution at the expense of increased algorithm complexity. Besides, in real scenario, it is necessary to consider the effect of source terms such as the bed slope and friction slope. Compared with other high-resolution shock-capturing schemes, TVD-MacCormack-type predictor–corrector-updating method is easy to implement and does not bring in any additional difficulty in dealing with the source terms [7–10].

As we know, most of the flow routing models have some numerical difficulties, if the irregular bottom topography is present, especially, very poor results are usually obtained if most of the high-resolution shock-capturing schemes are applied without special treatment to simulate the open-channel flow with strong bed slope variations. Recently, Bermúdez and Vázquez [11] and Vázquez-Cendón [12] proposed an upwind method to solve shallow water flow with source terms. LeVeque [13] developed a treatment for source terms balancing with flux gradients for a quasi-steady problem. Hubbard and Garcia-Navarro [14] suggested a method for balancing source terms and flux gradients based on the upwind approach of Bermúdez and Vázquez. However, these upwind methods are complex. Zhou *et al.* [15] proposed the surface gradient method for shallow water equations with source terms such as bed slope, and the bed slope term is discretized with a centred scheme. To eliminate or reduce the artificial numerical error caused by strong channel bed slope variations for TVD-MacCormack scheme, three improved approaches on the simulation of free surface flows in channel with high irregular bed topography are developed and compared in this paper.

2. MATHEMATICAL FORMULATION

Based on the assumption of hydrostatic pressure distribution and incompressible flows, the St Venant equations for one-dimensional, unsteady open-channel flows can be described as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S} \tag{1}$$

in which

$$\mathbf{Q} = \begin{pmatrix} A \\ Q \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} Q \\ Q^2 A^{-1} + gI_1 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ gA(S_\mathrm{o} - S_\mathrm{f}) \end{pmatrix}$$

where t represents the time, x is the longitudinal distance along a channel, A is the wetted cross-sectional area, Q is the volume flow rate, g is the gravitational acceleration, I_1 is the hydrostatic pressure force, S_o is the bed slope, and S_f is the friction slope. In this study, flows through a prismatic open-channel of bed slope and friction slope are modelled to illustrate the use of different treatments for these source terms.

Equation (1) can be expressed in quasi-linear form as

$$\frac{\partial \mathbf{Q}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{Q}}{\partial x} = \mathbf{S} \tag{2}$$

where **A** is the Jacobian matrix having two real eigenvalues

$$\lambda_1 = u + c, \quad \lambda_2 = u - c \tag{3}$$

where $u = Q/A$ is the flow velocity, $c = \sqrt{gA/T}$ is the wave celerity, and T is the wetted top width of the cross-section. In this study, the corresponding right and left eigenvector matrices for matrix **A** are defined as

$$\mathbf{R} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix}, \quad \mathbf{L} = \frac{1}{2c}\begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix} \tag{4}$$

For the hyperbolicity, the Jacobian matrix **A** can be found

$$\mathbf{A} = \mathbf{R\Lambda L}, \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{5}$$

3. NUMERICAL MODEL

3.1. Original TVD-MacCormack model

In view of the numerous advantages of MacCormack-type predictor–corrector scheme [16], it is still widely used in computational hydraulics. Firstly, the scheme is a shock-capturing technique with second-order accuracy both in time and space. Secondly, the inclusion of the source terms is relatively simple. And, thirdly, it is suitable for implementation in explicit time-marching algorithm. The computational domain is discretized as $x_j = j\Delta x$ and $t^n = n\,\Delta t$, where Δx is the size of a uniform mesh, and Δt is the time increment. The predictor–corrector three-step procedure is expressed as [7, 9, 10]

$$\text{Predictor step: } \mathbf{Q}_j^{\mathrm{p}} = \mathbf{Q}_j^n - \frac{\Delta t}{\Delta x}(\mathbf{F}_{j+1}^n - \mathbf{F}_j^n) + \Delta t\,\mathbf{S}_j^n \tag{6}$$

$$\text{Corrector step: } \mathbf{Q}_j^{\mathrm{c}} = \mathbf{Q}_j^n - \frac{\Delta t}{\Delta x}(\mathbf{F}_j^{\mathrm{p}} - \mathbf{F}_{j-1}^{\mathrm{p}}) + \Delta t\,\mathbf{S}_j^{\mathrm{p}} \tag{7}$$

$$\text{Updating step: } \mathbf{Q}_j^{n+1} = \frac{1}{2}(\mathbf{Q}_j^{\mathrm{p}} + \mathbf{Q}_j^{\mathrm{c}}) + \frac{1}{2}\frac{\Delta t}{\Delta x}\left(\mathbf{R}_{j+1/2}\mathbf{\Phi}_{j+1/2} - \mathbf{R}_{j-1/2}\,\mathbf{\Phi}_{j-1/2}\right) \tag{8}$$

where the superscript p and c denote the variables at predictor and corrector steps, respectively. The predictor step in Equation (6) is a forward difference in space; the corrector step in Equation (7) is a backward difference in space. Owing to the fact that MacCormack scheme incorporates forward and backward differences in separated predictor and corrector steps, four different combinations can be found for one-dimensional problems [9]. In this study, the forward–backward differences are rotated during the computation to avoid the accumulation of errors.

Equation (8) furnishes the scheme with total variation diminishing (TVD) dissipation, which is capable of rendering the solution oscillation free, while retaining second-order accuracy in space and time almost everywhere (except at extreme points) [1, 2]. This is an important feature that is capable of dealing with transcritical and rapidly varied flows such as hydraulic jumps and surges. To achieve this purpose, the component of $\mathbf{\Phi}_{j+1/2}$ in Equation (8) is defined as

$$\Phi^k_{j+1/2} = \psi(\lambda^k_{j+1/2}) \left(1 - \frac{\Delta t}{\Delta x}|\lambda^k_{j+1/2}|\right) (1 - \varphi(r^k_{j+1/2}))\alpha^k_{j+1/2} \tag{9}$$

The entropy fix function ψ is

$$\psi(z) = \begin{cases} |z| & \text{if } |z| \geqslant \varepsilon \\ \varepsilon & \text{if } |z| < \varepsilon \end{cases} \tag{10}$$

where ε is a small positive number whose value has to be determined for each individual problem. Harten and Hyman [17] suggested a formula to calculate ε in order to cut down trial process

$$\varepsilon^k_{j+1/2} = \max[0,\ \lambda^k_{j+1/2} - \lambda^k_j,\ \lambda^k_{j+1} - \lambda^k_{j+1/2}] \tag{11}$$

The characteristic variable α is

$$\alpha_{j+1/2} = \frac{1}{2c_{j+1/2}} \begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix}_{j+1/2} \cdot \begin{bmatrix} A_{j+1} - A_j \\ Q_{j+1} - Q_j \end{bmatrix} \tag{12}$$

The subscript $(j + \frac{1}{2})$ denotes the intermediate state between grid points j and $(j + 1)$.

Following the technique suggested by Roe [3], the mean values of velocity and wave celerity can be calculated as

$$u_{j+1/2} = \frac{\sqrt{A_j}u_j + \sqrt{A_{j+1}}u_{j+1}}{\sqrt{A_j} + \sqrt{A_{j+1}}}, \quad c_{j+1/2} = \sqrt{\frac{gA_{j+1/2}}{T_{j+1/2}}} \tag{13}$$

The purpose of the flux limiter function φ in Equation (9) is to supply artificial dissipation when there is a discontinuity or a strong gradient, while adding very little or no dissipation in regions of smooth variation. There are several forms of the function φ suggested in References [1–3]. In this study, a minmod limiter of function φ was used

$$\varphi(r^k_{j+1/2}) = \begin{cases} \min(|r^k_{j+1/2}|, 1), & r^k_{j+1/2} > 0 \\ 0, & r^k_{j+1/2} \leqslant 0 \end{cases} \tag{14}$$

where

$$r^k_{j+1/2} = \frac{\alpha^k_{j+1/2-s}}{\alpha^k_{j+1/2}}, \qquad s = \text{sign}(\lambda^k_{j+1/2}) \tag{15}$$

The source terms appearing on the right-hand side of Equations (6) and (7) are evaluated as follows:

$$(S_o)_j = \frac{z_{j-1} - z_{j+1}}{2\Delta x}, \qquad (S_f)_j = \frac{n^2 Q_j |Q_j|}{A_j^2 R_j^{4/3}} \tag{16}$$

where z is the elevation of the bed, R is the hydraulic radius and n is the Manning coefficient.

For the stability of explicit scheme, the Courant–Friedrich–Lewy condition must be satisfied

$$\Delta t = Cr\left[\frac{\Delta x}{|u| + c}\right] \tag{17}$$

where Cr is the Courant number. In this study, the value of Cr can be set to 1.0, usually 0.95. The convergence criterion for a steady solution is defined as the sum of the relative error of water depth at every grid between two time levels less than 5×10^{-6}.

The finite-difference TVD-MacCormack scheme developed in this study is devised for interior points. Boundary conditions are implemented by giving phantom grids outside the computational domain where dependent variables and gradients are specified. For subcritical flows, velocity is needed at the inflow boundary, and water depth has to be specified at the outflow boundary. For supercritical flows, only the inflow boundary conditions such as the water depth and velocity are needed, and none at the outflow boundary. The remaining unknown variables are provided by giving a zero gradient condition at the boundaries.

3.2. Model test

In this section, the original TVD-MacCormack model outlined above is tested by solving two benchmark problems including steady smooth and transcritical flows in a channel with irregular bed topography.

3.2.1. Steady smooth flow in irregular bed topography channels. To demonstrate the original model to solve the steady smooth flow in channels with strong variations of bottom topography, a steady flow over an irregular bed was proposed at a workshop on dam break wave simulations [18]. This case gives a good insight into the behaviour of numerical error; because of the steady-state flow conditions the discharge should be constant at any grid over the computational domain. The length of the channel is 1500 m long, the Manning coefficient is 0.1, and the irregular bed topography of the channel is shown in Figure 1(a). The same bed is also used by V'azquez-Cend'in [12], Hubbard and Garcia-Navarro [14] and Zhou *et al.* [15]. At the upstream end, a constant discharge of $0.75\,\mathrm{m}^2/\mathrm{s}$ is imposed, while the downstream water depth is fixed at the value of 15 m. In the computation, 201 grids with $\Delta x = 7.5$ m, which is the same as that by Vázquez-Cendón [12] and Zhou *et al.* [15].

Results given in Figure 1(a)–1(c) exhibit the variations of simulated water surface elevation, flow velocity and discharge along the channel for the original TVD-MacCormack model. It shows that the water surface elevation along the channel has some numerical errors, and the profiles of flow velocity and discharge have more unphysical oscillations. For a quantitative comparison with the mass conservation characteristic of the numerical scheme, the relative errors are defined by the Qerr and Herr indexes

$$\mathrm{Qerr} = \left(\frac{Q_j - Q_\mathrm{u}}{Q_\mathrm{u}}\right) * 100\%, \quad \mathrm{Herr} = \left(\frac{H_j - H_\mathrm{u}}{H_\mathrm{u}}\right) * 100\% \tag{18}$$

where (Q_j, H_j) and $(Q_\mathrm{u}, H_\mathrm{u})$ are the discharge and water surface elevation at any grid j and at the upstream end. The results simulated by the original TVD-MacCormack model show that the value of Qerr is between -119 and 103%, and the value of Herr is between -0.6 and 0.8% for this irregular bottom topography case. To obtain accurate solutions without excessive grid refinement, the result indicates that it is necessary to develop a special treatment of

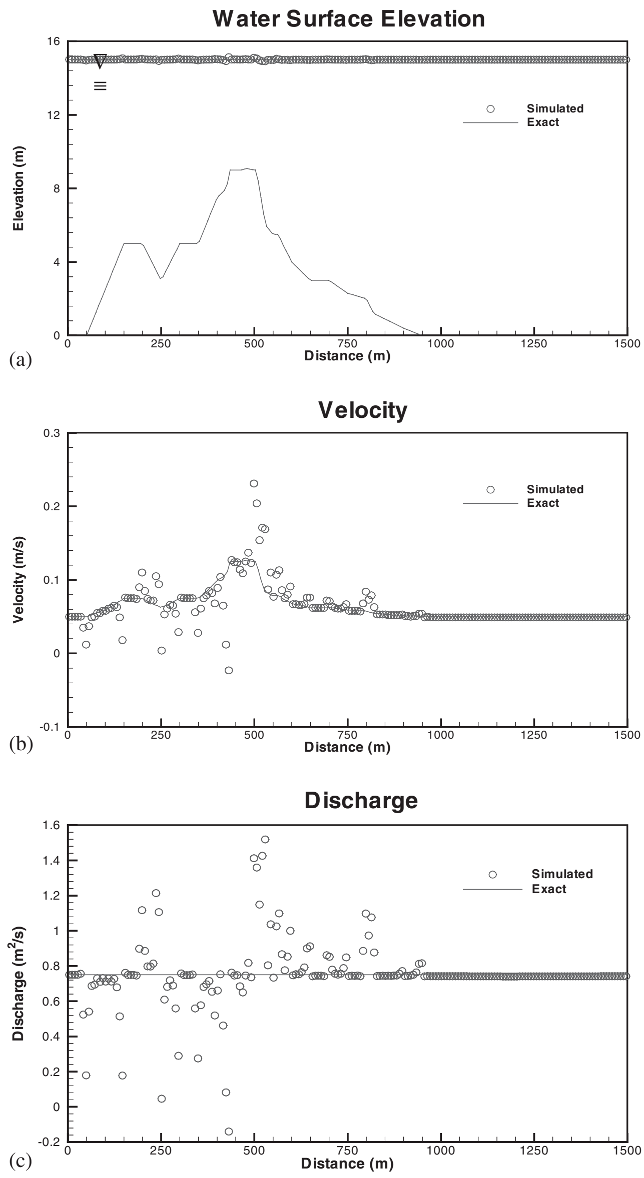

Figure 1. Steady smooth flow with irregular bed topography (original model): (a) Water surface elevation; (b) Velocity and (c) Discharge.

source terms to avoid the generation of artificial numerical error due to irregular bed elevation variations even in the smooth flow regime.

3.2.2. Steady transcritical flow in irregular bed topography channel. Another very interesting case presented in this section is the steady transcritical flow in strongly varying bottom

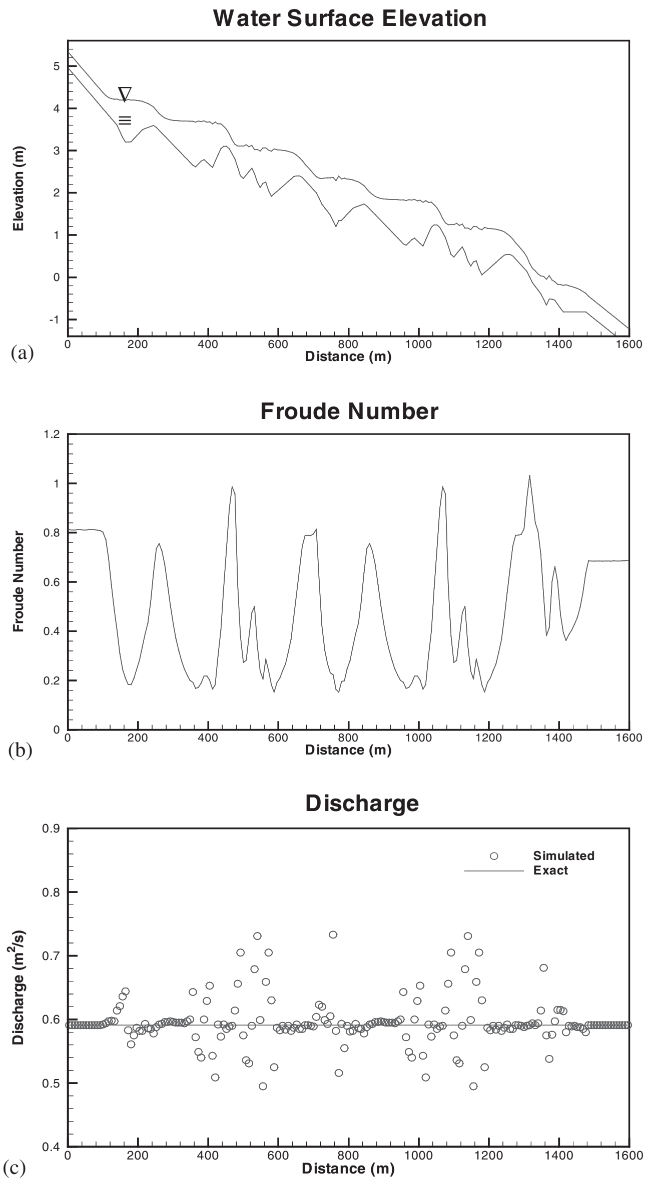

Figure 2. Steady transcritical flow with irregular bed topography (original model): (a) Water surface elevation; (b) Froude number and (c) Discharge.

topography, which to the writers' knowledge, has not been reported before. This configuration is commonly encountered in engineering practice in some natural mountain rivers. The length of the channel is 1600 m long, the Manning coefficient is 0.033, and the bottom topography of the channel is shown in Figure 2(a). At the upstream end, a constant discharge of 0.59 m^2/s

is imposed, while the downstream water depth is fixed at the value of 0.42 m. A uniform grid size $\Delta x = 8$ m is used in the simulation.

Figure 2(a)–2(c) displays the variations of simulated water surface elevation, Froude number and discharge along the channel. It shows that the original TVD-MacCormack model can simulate this transcritical flow without any numerical difficulty. But it also illustrates that the mass conservation characteristic is poor due to the value of Qerr is between −16 and 24% for this irregular bottom topography case. The result shows once more that a special treatment of source terms is needed to reduce the artificial numerical error because of irregular bed topography which is likely in nature.

4. IMPROVEMENT OF MASS CONSERVATION

From the results of a large number of tests, it is evident that some special numerical treatment is needed to insure mass conservation in the numerical solution of free surface flows in channels with irregular bed topography. For this purpose, three improved approaches have been studied. They are presented below.

4.1. Approach 1

Based on the finding of LeVeque [13] and Hubbard and Garcia-Navarro [14], the balancing of source terms and flux gradients is an important way to reduce the artificial numerical oscillations due to strong irregular bed elevation variations. In this formulation, the terms of bed slope and friction slope are discretized at each step with forward or backward differences in the same manner as the flux gradient term ($\partial \mathbf{F}/\partial x$) as follows:

$$\text{Predictor step: } (gAS_o)_j = gA_{j+1/2}\frac{(z_j - z_{j+1})}{\Delta x}, \quad (gAS_f)_j = g\frac{n^2 Q_{j+1/2}|Q_{j+1/2}|}{A_{j+1/2}R^{4/3}_{j+1/2}} \tag{19}$$

$$\text{Corrector step: } (gAS_o)_j = gA_{j-1/2}\frac{(z_{j-1} - z_j)}{\Delta x}, \quad (gAS_f)_j = g\frac{n^2 Q_{j-1/2}|Q_{j-1/2}|}{A_{j-1/2}R^{4/3}_{j-1/2}} \tag{20}$$

where $A_{j\pm1/2}$ and $Q_{j\pm1/2}$ are the arithmetic mean of the wetted cross-sectional area and the discharge between grid points j and $(j \pm 1)$.

Figure 3(a)–3(c) illustrates the simulated water surface elevation, Froude number and discharge for the benchmark problem of smooth flow at Section 3.2.1, respectively, and the value of Qerr is between −125 and 101%, also the value of Herr is between −0.5 and 0.7%. The corresponding water surface elevation, Froude number and discharge for the benchmark problem of transcritical flow at Section 3.2.2 is shown in Figure 4(a)–4(c), respectively, and the value of Qerr is between −13 and 22%. These results obtained from this approach have clearly demonstrated that the mass conservation cannot be improved by comparing with Figures 1 and 3, and Figures 2 and 4. The outcome may be due to adding the TVD correction in flux gradient is incompatible with the source terms in this approach.

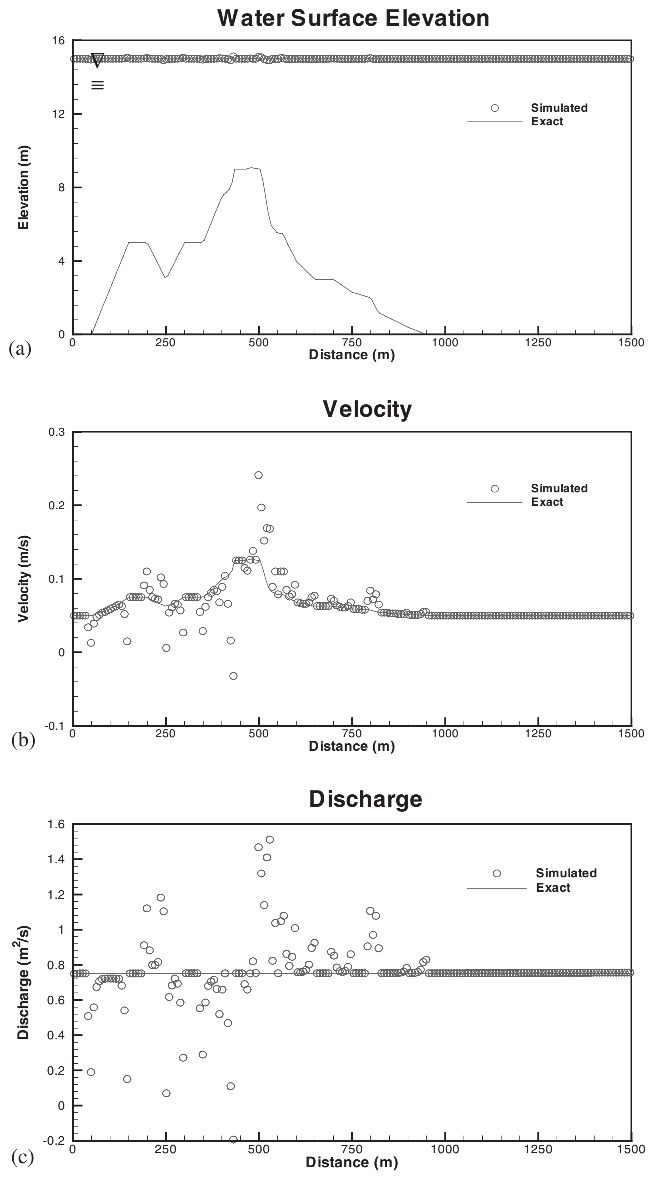

Figure 3. Steady smooth flow with irregular bed topography (improved approach 1): (a) Water surface elevation; (b) Velocity and (c) Discharge.

4.2. Approach 2

In general, the water surface profile or elevation H along the channel flow is normally much smoother than the bed elevation z or water depth h. The idea of surface gradient method [15] is used in this approach, where the water surface elevation is chosen to calculate the TVD

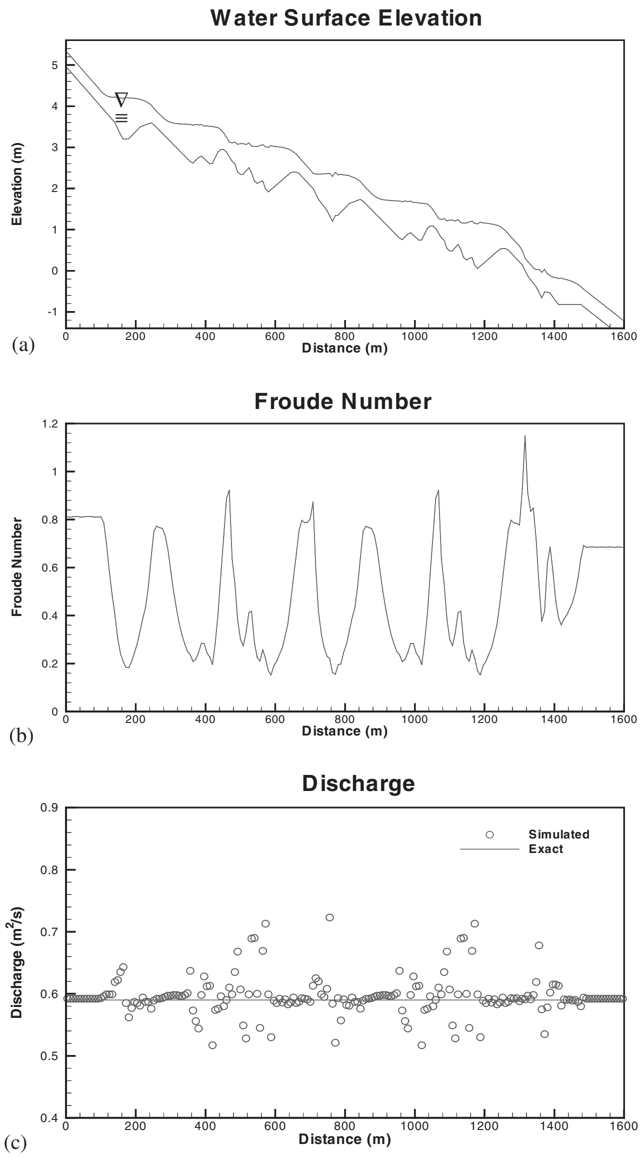

Figure 4. Steady transcritical flow with irregular bed topography (improved approach 1): (a) Water surface elevation; (b) Froude number and (c) Discharge.

correction. For the locally rectangular channel, Equation (12) can be rewritten as

$$\alpha_{j+1/2} = \frac{1}{2c_{j+1/2}} \begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix}_{j+1/2} \cdot \begin{bmatrix} h_{j+1} - h_j \\ (hu)_{j+1} - (hu)_j \end{bmatrix} \tag{21}$$

The only difference between the original TVD-MacCormack model and this improved approach is that the characteristic variable $\alpha_{j+1/2}$ in Equation (21) is replaced by

$$\alpha_{j+1/2} = \frac{1}{2c_{j+1/2}} \begin{bmatrix} -\lambda_2 & 1 \\ \lambda_1 & -1 \end{bmatrix}_{j+1/2} \cdot \begin{bmatrix} H_{j+1} - H_j \\ (hu)_{j+1} - (hu)_j \end{bmatrix} \tag{22}$$

This treatment is the same as the original TVD-MacCormack model in the absence of bed slope term.

Figure 5(a)–5(c) demonstrates the simulated water surface elevation, Froude number and discharge obtained by this approach for the benchmark problem of smooth flow, respectively, and the value of Qerr is between -9 and 11%, also the value of Herr is between -0.2 and 0.0%. The corresponding water surface elevation, Froude number and discharge for the benchmark problem of transcritical flow are shown in Figure 6(a)–6(c), respectively, and the value of Qerr is between -3 and 4%. By comparing with Figures 1 and 5, and Figures 2 and 6, it is seen that the numerical error of discharge is reduced by more than an order of magnitude for both smooth and transcritical flow test cases. However, it also indicates that the improved approach outlined above has to be further improved to eliminate or further reduce the artificial numerical error caused by strong channel bed slope variations.

4.3. Approach 3

According to the nature of TVD-MacCormack-type predictor–corrector-updating procedure, a hybrid approach is proposed by discretizing the source terms at each step in the same manner as the flux gradient incorporated using the water surface elevation instead of water depth for the TVD correction. This improved approach employs Equations (19) and (20) to discretize the source terms in Equations (6) and (7), and uses Equation (22) instead of Equation (12). This treatment just combined the improved strategy of approaches 1 and 2.

Results given in Figure 7(a)–7(c) exhibit the variations of simulated water surface elevation, Froude number and discharge along the channel for the benchmark test of smooth flow by the improved approach 3, respectively, the values of Qerr and Herr are eliminated exactly to -0.0 and 0.0%. The results obtained from this approach have evidently demonstrated that the mass conservation of the smooth open-channel flow can be preserved very well in every grid even in flows with high irregular bed topography. The corresponding water surface elevation, Froude number and discharge for the benchmark problem of transcritical flow are shown in Figure 8(a)–8(c), respectively, and the value of Qerr is between -2 and 3%. It is also seen that the mass conservation can be satisfied well by using this improved approach even in the transcritical flows with strong bed slope variations.

By comparing the results of Figures 1–8, the improved approach 3 is the most accurate, the improved approach 2 ranks second and the original TVD-MacCormack formulation is the least accurate. Besides, all of the three improved approaches involve an equivalent level implementation complexity and computation effort to the original TVD-MacCormack model. It can be seen that the improved approach 3 is more suitable for simulating open-channel flow with highly irregular bed topography by using the surface gradient instead of the depth gradient and considering the balancing of the source terms and the flux gradients. In the

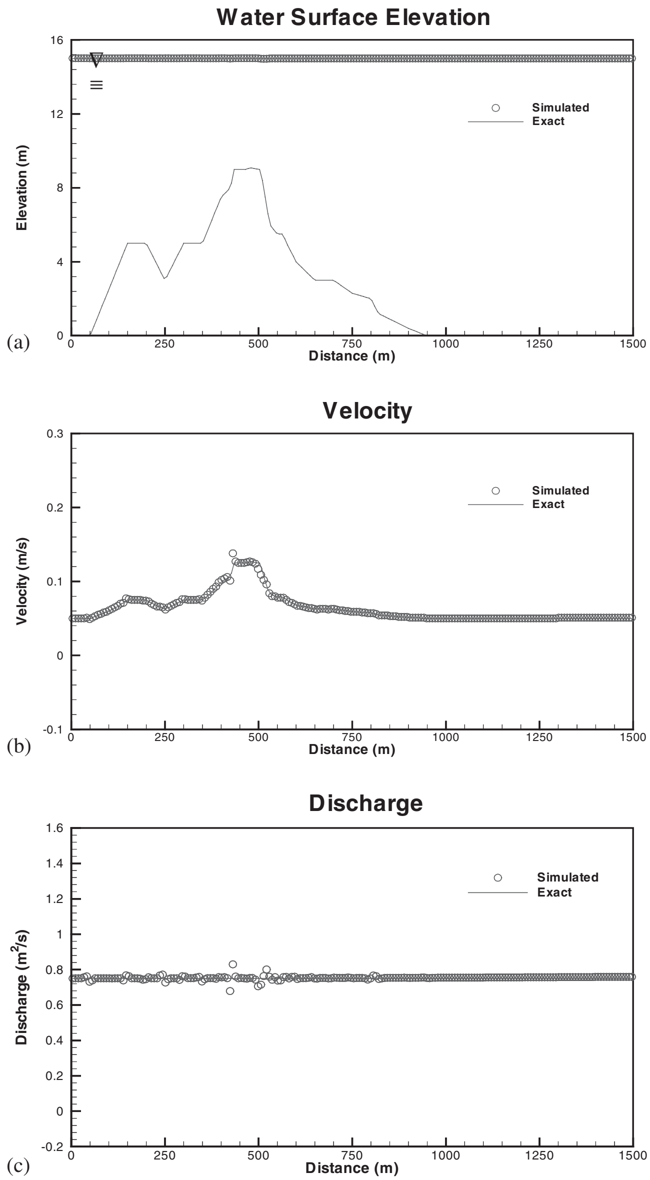

Figure 5. Steady smooth flow with irregular bed topography (improved approach 2): (a) Water surface elevation; (b) Velocity and (c) Discharge.

following, the proposed approach is verified by solving more benchmark problems including both steady and unsteady flows with source terms effect. The accuracy is shown by comparing the numerical solutions with analytical solutions, available numerical results and experimental data.

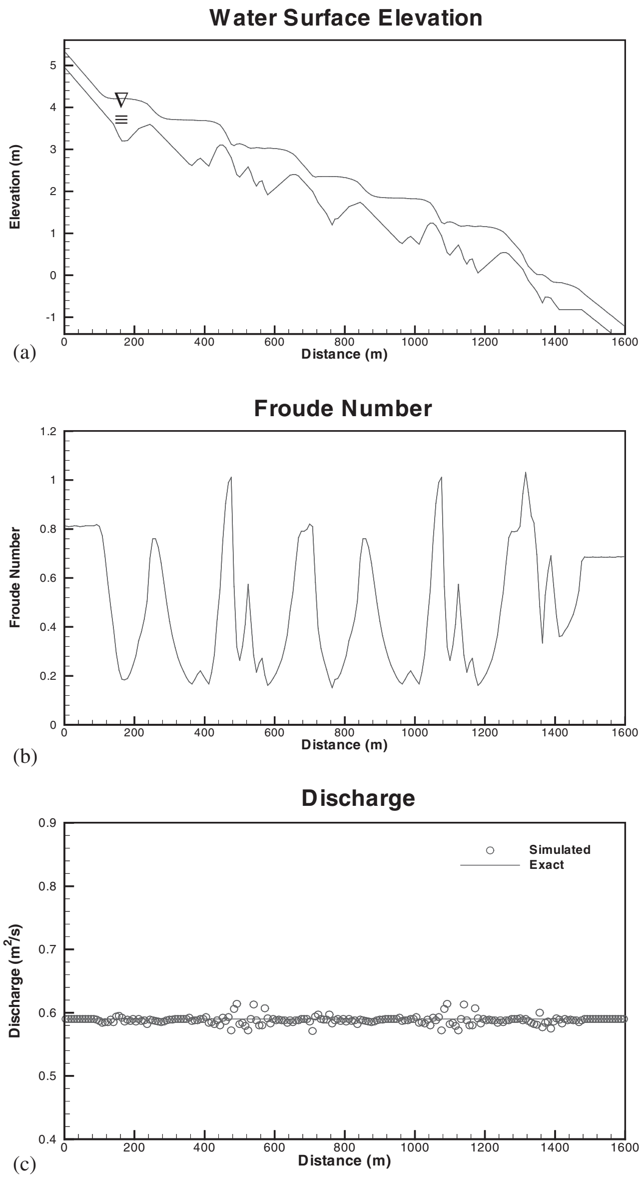

Figure 6. Steady transcritical flow with irregular bed topography (improved approach 2): (a) Water surface elevation; (b) Froude number and (c) Discharge.

4.4. Steady smooth flow over hump

This test problem is a steady frictionless flow with a bell-shaped hump at channel bottom. An analytical solution can be found for this case. This example is a simplified case that represents channel flows of irregular bottom topography. The bell-shaped hump starts at $x = 125$ m and

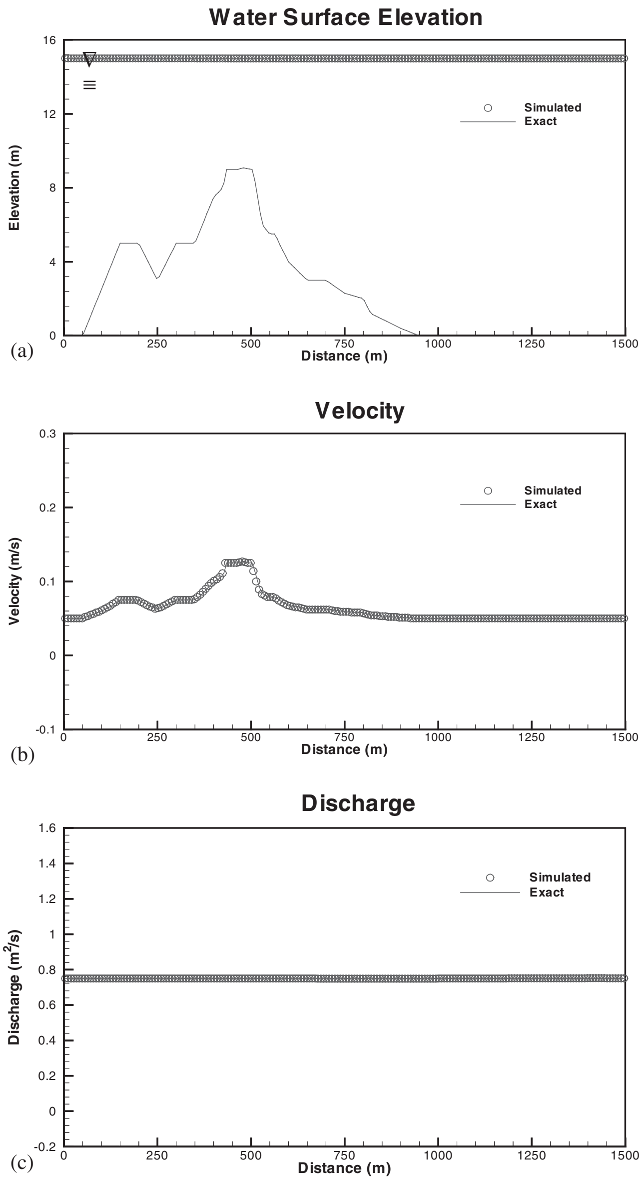

Figure 7. Steady smooth flow with irregular bed topography (improved approach 3): (a) Water surface elevation; (b) Velocity and (c) Discharge.

ends at $x = 875$ m, and the bed elevation z of the channel shown in Figure 9(a) can be described as

$$z(x) = 4.75 \sin^2\left(\frac{x - 125}{750}\pi\right) \tag{23}$$

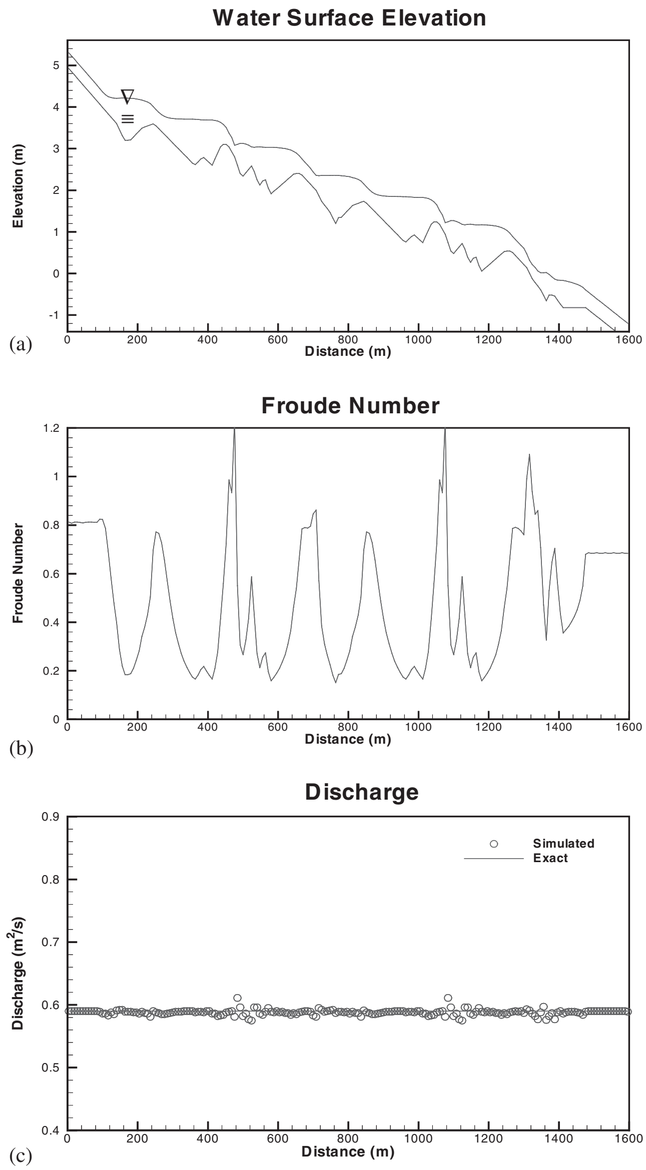

Figure 8. Steady transcritical flow with irregular bed topography (improved approach 3): (a) Water surface elevation; (b) Froude number and (c) Discharge.

At the upstream end, a discharge of $20.0\,m^2/s$ is imposed and no boundary condition was needed at the downstream end of the channel. These conditions lead to a hydraulic drop near the hump. An analytical solution can be derived from the conservation of mass and energy. A uniform grid distribution with 101 grids is used in the computation. Figure 9(a) and 9(b) compares the simulation results of water surface elevation and Froude number with

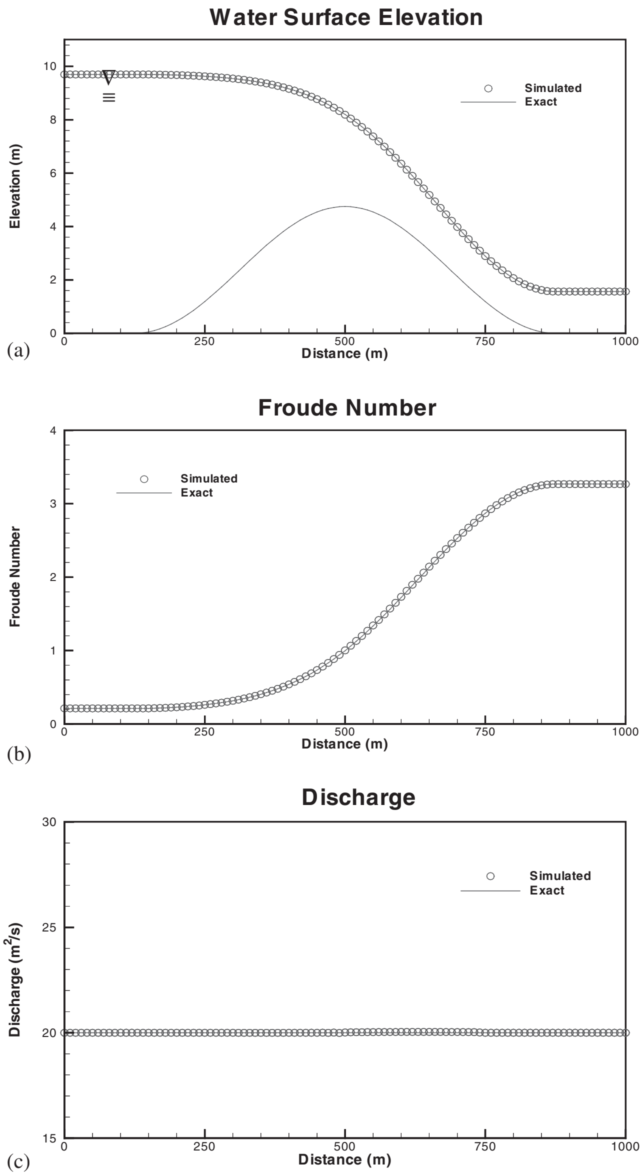

Figure 9. Steady smooth flow over hump: (a) Water surface elevation; (b) Froude number and (c) Discharge.

the analytical solutions, respectively. As can be seen, the agreement between the analytical solution and the numerical solution is remarkable. Figure 9(c) shows a variation of discharge along the channel, and an excellent mass conservation characteristic of the proposed approach is preserved even for the channel flows with the bell-shaped bottom topography.

4.5. Steady transcritical flow over hump

In this case, a discharge of $20.0\ m^2/s$ is imposed at the upstream end, while the downstream water depth is fixed to the value of 7.0 m. These conditions lead to a hydraulic jump near the hump. An analytical solution can be derived from the conservation of mass and energy combined with the specific-force relation. The simulation results of water surface elevation, Froude number and discharge compared with the analytical solutions are shown in Figure 10(a)–10(c), respectively. The numerical solutions are obtained by using 101 uniform grids. The agreement between the analytical solution and the numerical solution is very well. The results compare favourably with those of previous papers [7, 15, 19]. This comparison indicates that the proposed approach can handle the transcritical flows automatically, and the hydraulic jump is accurately captured without any oscillation.

4.6. Steady transcritical flow in rough sloping channel

The flow configuration in this test represents transcritical flows in a rectangular channel consisting of 4 reaches with different bed slopes ($S_{o1} = 0.03$, $L_{o1} = 40\,m$; $S_{o2} = 0.008$, $L_{o2} = 40\,m$; $S_{o3} = 0.03$, $L_{o3} = 40\ m$; $S_{o4} = 0.011$, $L_{o4} = 40\ m$). The total length of the channel is 160 m. The discharge is set to be $2.36\ m^2/s$ at the upstream and the water depth is set equal to 0.84 m at downstream boundary, the Manning's coefficient is 0.033, and the grid size is 1 m. The water surface and Froude number simulated by the proposed approach are represented in Figure 11. The locations of the hydraulic jumps and hydraulic drop can be accurately described by the proposed approach without any oscillation. The result compares favourably with that of Tseng *et al.* [19]. Figure 11(c) shows the variation of discharge along the channel, and an excellent mass conservation characteristic of the proposed approach is preserved even for the transcritical flows in the rough sloping channel.

4.7. Dam break flow in rough sloping channel

The above cases only test the proposed approach to simulate open-channel flow in steady state. In order to demonstrate that the proposed scheme is capable of describing unsteady flow, a laboratory dam-break experiment of Waterway Experiment Station (WES), U.S. Corps of Engineers [20] is also simulated in this study. The experiment was conducted in a rectangular channel with length of 122 m, width of 1.22 m, bottom slope of 0.005, and the Manning coefficient $n = 0.045$. The water depth upstream of the dam is 0.305 m, and the downstream water depth is zero. The flow domain is discretized into 123 grids with a uniform grid spacing $\Delta x = 1.0$ m. Figure 12(a) and 12(b) shows a comparison of the computed and measured water surface profiles at four different locations. Figure 12(a) compares the simulated water surface profiles with the experimental data at 39.62 and 6.10 m upstream of the dam, while Figure 12(b) exhibits the water surface profiles at 7.62 and 45.72 m downstream of the dam. The proposed approach predicts well the arrival time for positive and negative waves and water surface elevation both upstream and downstream of the dam for sloping channel with high hydraulic resistance.

5. CONCLUDING REMARKS

Four different treatments to discretize the source terms, including the original TVD-MacCormack formulation and three improved approaches, are presented in this paper for

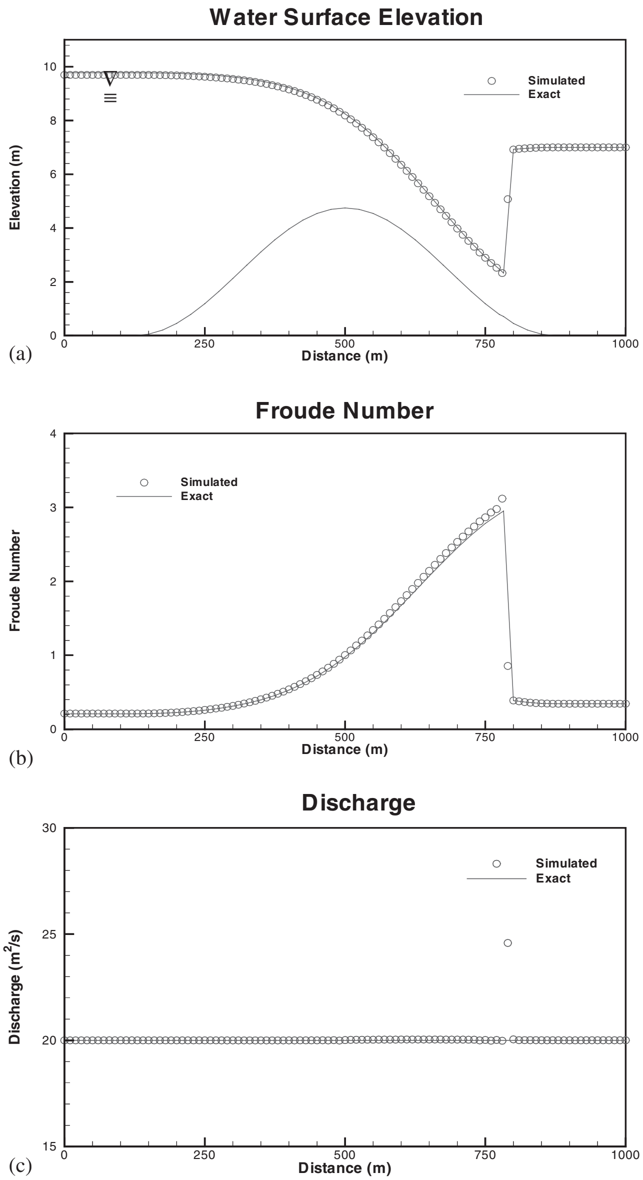

Figure 10. Steady transcritical flow over hump: (a) Water surface elevation; (b) Froude number and (c) Discharge.

the computation of one-dimensional open-channel flows with rapidly varied bed topography by using the finite-difference TVD-MacCormack scheme.

In the original TVD-MacCormack formulation, a centred difference is employed to discretize the bed slope term and a pointwise method is applied for the friction slope term. Based on solving two benchmark problems including steady smooth and transcritical flows, the simulated

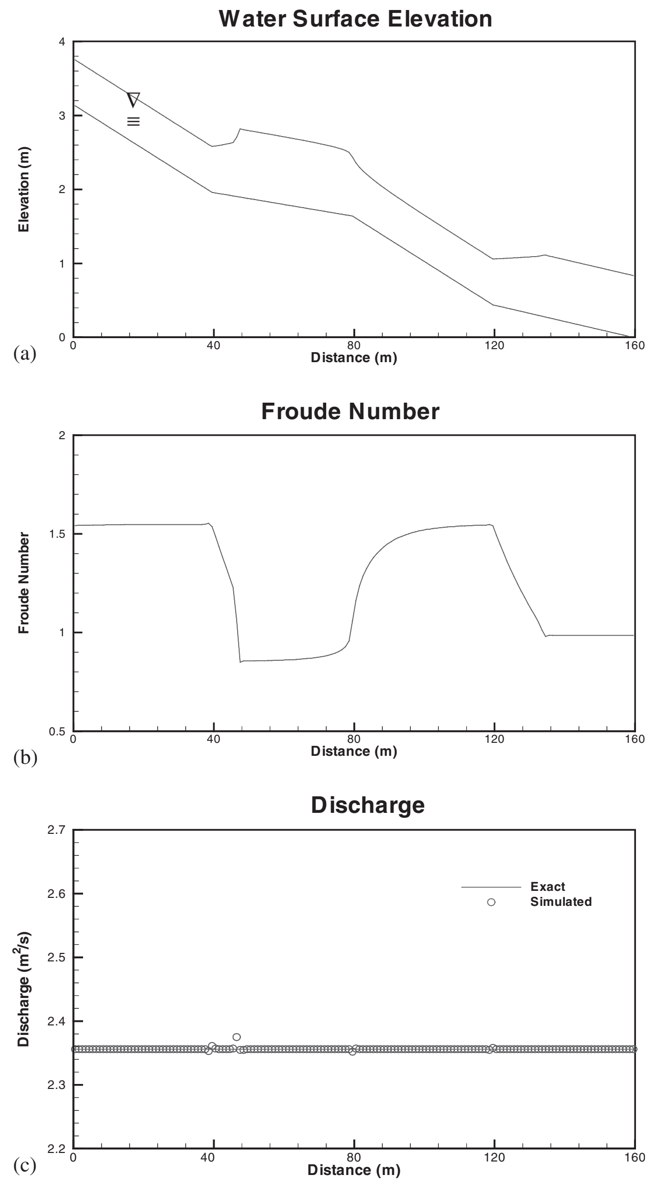

Figure 11. Steady transcritical flow in rough sloping channel: (a) Water surface elevation; (b) Froude number and (c) Discharge.

results obtained by the original TVD-MacCormack model show that a special treatment of source terms is needed to reduce the artificial numerical error caused by strong channel bed slope variations.

In the improved approach 1, the terms of bed slope and friction slope are discretized at each step with forward or backward differences in the same manner as the flux gradient term

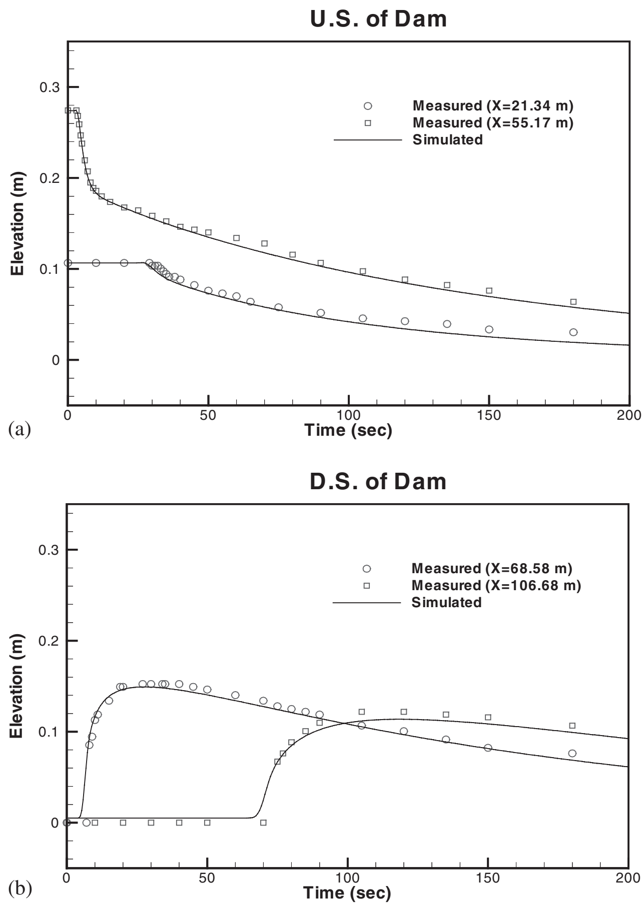

Figure 12. Dam break flow in rough sloping channel: (a) U.S. of Dam and (b) D.S. of Dam.

to achieve balancing between flux gradients and source terms. In the improved approach 2, the water surface elevation instead of water depth is chosen to calculate the TVD correction, i.e. the surface gradient method (SGM). In the improved approach 3, a hybrid treatment is proposed by combining approaches 1 and 2 together, which is referred to as the improved SGM. By comparing the results obtained by the three improved approaches and the original TVD-MacCormack model, the improved SGM is the most accurate, the SGM ranks second and the original TVD-MacCormack formulation is the least accurate. Besides, all of the three improved approaches involve an equivalent level implementation complexity and computation effort to the original TVD-MacCormack model. It can be seen that the improved SGM is more suitable for simulating open-channel flow with highly irregular bed topography by using the surface gradient instead of the depth gradient for TVD corrections, considering the balancing of the source terms and the flux gradients.

Finally, verifications of the proposed improved approach are made by comparison with analytical solutions or available experimental data for both steady and unsteady problems involving the source terms effect, and very good agreements are found. Furthermore, results of the proposed improved SGM approach exhibit high accuracy, robustness, stability, simplicity and efficiency for simulation of channel flows with strong bed elevation variations.

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